VARIANCE ESTIMATION OF CHANGES IN OVERLAPPING SAMPLES: AN APPLICATION TO THE ITALIAN SURVEY ON SERVICE TURNOVER[1]

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1. Introduction

This work was inspired by the growing need to have a measure of the accuracy of the estimates produced within the short-term statistics in the Official Statistics. In particular, the aim of the work is to illustrate the methodology for the computation of the variance for the estimators currently used in the Italian service turnover survey (ISTS, for brevity) carried on by the Italian National Institute of Statistics (ISTAT) for the quarterly turnover growth rate estimation. While the calculation of the variance of the estimates produced for a given instant of time is now a good practice (also through the development of software packages), the same does not happen for the variation of two quantities over time. An estimator of variance must take into account of both the estimator and the sampling design (Wolter, K.M. (1985)). The greatest difficulty is that for many surveys, the samples for producing estimates in two different time are not independent each other, due to the rotation operations of the sample. In particular for business surveys, in order to take into account the birth-mortality of units in the population and changes in stratification variables (such as size category and type of economic activity), the sample is updated, and a part of the units is replaced with others. This means that in calculating the estimate of the variance of change over time, we need not only the estimates of the variances of the cross-sectional estimates, but also the covariance terms between cross-sectional estimates. Moreover, many indicators are non-linear function of linear estimators (e.g. simple ratio, difference of ratios), therefore, to calculate their variance a first-order Taylor approximation can be used. This is the case, for example, for the variance estimations of the LFS-based indicators' annual net changes (Ceccarelli *et al.*, (2017)). Alternatively, balanced repeated replication (BRR) can be used (Moretti *et al.*, (2005)). The variance for the estimators

[1] This paper builds on the work done in the context of my Ph.D thesis. I wish to express my gratitude to Pierluigi Conti for the supervision of the thesis and Pier Francesco Perri and Emilia Rocco for their suggestions. The views expressed in this paper are solely those of the author and do not necessarily reflect Istat official positions. The full version of the thesis paper is available at https://iris.uniroma1.it/handle/11573/1315826

currently used in the service turnover survey is computed only for the total estimations in the quarters t and t-4, while the variance of the growth rate estimation for the different estimation domains is not calculated. The aim of the present paper is not only to suggest how to assess the variance of possible estimators of the turnover variation over time, but also to compare such estimators with respect to their variance to identify the best one.

2. Description of the survey: sampling design and method of estimation

The ISTS[2] uses a stratified simple random sampling without replacement. The auxiliary information for the planning of the design is contained in the Istat Statistic Register of Active Firms (ASIA). ASIA is a register of enterprises and local units updated annually by Istat through a process of integrating administrative and statistical sources. It includes all economic units in industry and service sectors and provides identifying and structure information of these units. The information in it contains a time lag of two years. Each year, the sample for the ISTS is updated to account for both a re-stratification of the units and a sample replacement of approximately 15%. The units in the sample are re-stratified according to their actual size and economic activity from ASIA. Dead companies are discarded from the sample, together with the companies that have been in the sample for several years. New companies are randomly selected from the last ASIA available, excluding the units already in the sample (plan A of Tam, 1984), until the theoretical size provided by the Mauss-R software (see Barcaroli *et al.,* 2010), is reached within each stratum. New companies entering in the sample are required to indicate the turnover data for both the current year (t) and the previous year (t-4). In this way, it is possible to have turnover data for both estimation quarters, even if the firm was not in the sample at the occasion t-4. The estimates of the change between the occasion t and the occasion t-4 are both computed on the new sample updated to the last year (Chianella *et al.,* 2015). It means that all observations are stratified in the same way over the two estimation quarters, according to the latest information available on the stratification variables. The rotated units are not included in the estimates, neither in the quarter t nor in the quarter t-4. Let g the unknown growth rate for the turnover in the population:

$$g = (G - 1) = \left(\frac{Y_t}{Y_{t-4}} - 1\right)$$

Four estimators of $\hat{G}$ are presented for the estimate of the year-over-year growth rate of the turnover (Table 1).

[2] You can find useful information about the survey and the methodological note here: https://www.istat.it/it/archivio/fatturato+services

Table 1 – *Estimators used for the turnover growth rate estimation.*

Estimator of G	All respondent units	Only overlapping respondent units
Ratio of sample means	$\hat{G}_{all}$	$\hat{G}_{olp}$
Ratio of estimated totals	$\hat{G}_{all.cal}$	$\hat{G}_{olp.cal}$

Let r_1 be the set of the respondent enterprises only at the occasion t-4, r_2 the set of respondent enterprises in overlap between the occasions t-4 and t, r_3 the set of respondent enterprises only at the occasion t. Then we define $r_{12} = r_1 \cup r_2$ and $r_{23} = r_2 \cup r_3$:

1. $\hat{G}_{olp}$ is based on the ratio of the sample means calculated by using turnover data on the overlapping respondent units (r_2) between the two quarters:

$$\hat{G}_{olp} = \frac{\hat{Y}_{olp}^{t}}{\hat{Y}_{olp}^{t-4}} = \frac{\hat{\bar{y}}_{r_2}^{t}}{\hat{\bar{y}}_{r_2}^{t-4}}$$

2. $\hat{G}_{all}$ is based on the ratio of the sample means calculated using turnover data on all respondent units over the two quarters:

$$\hat{G}_{all} = \frac{\hat{Y}_{all}^{t}}{\hat{Y}_{all}^{t-4}} = \frac{\hat{\bar{y}}_{r_{23}}^{t}}{\hat{\bar{y}}_{r_{12}}^{t-4}}$$

3. $\hat{G}_{olp.cal}$ is based on the ratio of the estimated total of the turnover for the quarter t and for the quarter t-4, calculated using turnover data on the overlapping respondent units between the two quarters and through calibration (Deville and Sarndal, 1992) of the design weights:

$$\hat{G}_{olp.call} = \frac{\hat{Y}_{olp.cal}^{t}}{\hat{Y}_{olp.cal}^{t-4}} = \frac{\sum_{j\in r_2} y_j^t w_j}{\sum_{i\in r_2} y_i^{t-4} w_i}$$

4. $\hat{G}_{all.cal}$ is based on the ratio of the estimated total of the turnover for the quarter t and the quarter $t-4$, calculated using turnover data on all respondent units over the two quarters and through calibration of the initial weights:

$$\hat{G}_{all.call} = \frac{\hat{Y}_{all.cal}^{t}}{\hat{Y}_{all.cal}^{t-4}} = \frac{\sum_{j\in r_{23}} y_j^t w_j}{\sum_{i\in r_{12}} y_i^{t-4} w_i},$$

the calibrated weights (w_j and w_i) associated with the same unit on the two survey occasions of investigation (t and $t-4$) can be different due to the different non-response on the two occasions (the sets of respondent enterprises r_{12} and r_{23} are not the same). The ISTS uses for some domain estimations the estimator $\hat{G}_{olp}$ while for others the estimator $\hat{G}_{all.call}$ (Chianella *et al.*, 2013, Bacchini *et al.*, 2014, Bacchini *et al.*, 2015). In this work the estimators $\hat{G}_{olp.cal}$ and $\hat{G}_{all}$ are also analysed. The calibration variable used in the estimators $\hat{G}_{all.call}$ and $\hat{G}_{olp.call}$ is the annual turnover, due to its high correlation with the variable of interest. The values of the calibration variable and the known totals are the same in both the numerator and the denominator, and derive from the latest available Asia together with integration on sample data. Calibration is performed at single stratum level, i.e. the known totals are calculated for each stratum.

3. Which is the best estimator?

To decide which estimator has to be used, it is necessary to analyze their variance. The estimators presented in Section 2, are ratios between two estimates at different occasions (for brevity, let us denote them in a general way by $\hat{G} = \hat{Y}^t / \hat{Y}^{t-4}$). Since $\hat{G}$ is a non-linear function of linear estimators, the computation of his variance can be performed using the Taylor series approximation. The result that is obtained is as follows:

$$Var(\hat{G}) = \frac{1}{(Y^{t-4})^2}\left\{Var(\hat{Y}^t) + G^2 Var(\hat{Y}^{t-4}) - 2G cov(\hat{Y}^{t-4}, \hat{Y}^t)\right\}.$$

The variance terms in the above equation correspond to the variance of the total estimator (within the $\hat{G}_{olp}$ and $\hat{G}_{all}$ estimators we multiplied numerator and denominator by the number of units in the population (N) from which the sample is extracted). The difference between $Var(\hat{G}^{olp})$ and $Var(\hat{G}^{all})$, lies in the different number of units involved in the estimation: in the case of the $\hat{G}_{olp}$ estimator, only the number of units in overlapping between t and t-4 (n_c) is considered. We have:

$$Var(\hat{Y}^q_{all}) = N^2 Var(\hat{\bar{y}}^q_{all}) = N^2\left(\frac{1}{n} - \frac{1}{N}\right) S^2_{1Y} \ ,$$

$$Var(\hat{Y}^q_{olp}) = N^2 Var(\hat{\bar{y}}^q_{olp}) = N^2\left(\frac{1}{n_c} - \frac{1}{N}\right) S^2_{2Y} \ .$$

$Var(\hat{Y}^q_{all})$ and $Var(\hat{Y}^q_{olp})$ can be estimated substituting S^2_{1Y} and S^2_{2Y} (calculated on the entire population) with their estimate, calculated on the extracted sample. When using the $\hat{G}_{all.cal}$ and $\hat{G}_{olp.cal}$ estimators, the estimation of the total turnover is computed by calibration estimator. In this case, the variance of the total turnover in the generic quarter q can be approximated to the variance of the generalized regression model (see Righi *et al.*, 2005). Denoting by $z_j = y_j - x_j\beta$ the residuals of a regression model of Y (quarterly turnover data) on X (calibration variable), we define Z as the estimator of the total residuals. Therefore, we can write:

$$Var(\hat{Y}^q_{all.cal}) \cong Var(\hat{Z}^q_{all}) = N^2\left(\frac{1}{n} - \frac{1}{N}\right) S^2_{1Z} \ ,$$

$$Var(\hat{Y}^q_{olp.cal}) \cong Var(\hat{Z}^q_{olp}) = N^2\left(\frac{1}{n_c} - \frac{1}{N}\right) S^2_{2Z}$$

S_{1Z}^2 and S_{2Z}^2 represent the variance of the residuals computed on the population. An estimate of $Var(\hat{Y}_{all.cal}^q)$ and $Var(\hat{Y}_{olp.cal}^q)$ can be given estimating S_{1Z}^2 and S_{2Z}^2 on the sample observations:

$$\hat{S}_{1Z}^2 = \frac{1}{n-1}\sum_{i\in s}(z_i^q g_i^q - \bar{\tilde{z}}^q)^2\,; \quad \hat{S}_{2Z}^2 = \frac{1}{n_c-1}\sum_{i\in s}(z_i^q g_i^q - \bar{\tilde{z}}^q)^2$$

where g_i is the design weight correction factor associated with the i-th unit in the calibration process and $\tilde{z}$ is the mean of $z_i g_i$. For the computation of the covariance term, under the assumption of a fixed population (*N*), sample size (*n*) and overlapping rate between the two occasions ($o = n_c/n$), as well as of the same stratification (h) over time, the results of Tam (1984) and Qualité and Tillé (2008) have been easily derived (Andersson *et al.,* 2011). We obtain:

$$Cov(\hat{Y}_{all}^{t-4}, \hat{Y}_{all}^t) = N^2\left(\frac{o}{n} - \frac{1}{N}\right) S_{Y^t,Y^{t-4}}$$

$$Cov(\hat{Y}_{olp}^{t-4}, \hat{Y}_{olp}^t) = N^2\left(\frac{1}{n_c} - \frac{1}{N}\right) S_{Y^t,Y^{t-4}}$$

$$Cov(\hat{Y}_{all.cal}^{t-4}, \hat{Y}_{all.cal}^t) \cong Cov(\hat{Z}_{all.cal}^{t-4}, \hat{Z}_{all.cal}^t) = N^2\left(\frac{o}{n} - \frac{1}{N}\right) S_{Z^t,Z^{t-4}}$$

$$Cov(\hat{Y}_{olp.cal}^{t-4}, \hat{Y}_{olp.cal}^t) \cong Cov(\hat{Z}_{olp.cal}^{t-4}, \hat{Z}_{olp.cal}^t) = N^2\left(\frac{1}{n_c} - \frac{1}{N}\right) S_{Z^t,Z^{t-4}}.$$

The covariance within each estimation domain can be computed as the sum of the covariance calculated in the individual strata, due to the hypothesis of the same stratification over time. An estimation of $Cov(\hat{Y}_{all}^{t-4}, \hat{Y}_{all}^t)$ and $Cov(\hat{Y}_{olp}^{t-4}, \hat{Y}_{olp}^t)$ can be given estimating by the sample, the covariance between the quarters on the overlapping observations. Similarly, an estimate of $Cov(\hat{Y}_{all.cal}^{t-4}, \hat{Y}_{all.cal}^t)$ and $Cov(\hat{Y}_{olp.cal}^{t-4}, \hat{Y}_{olp.cal}^t)$ can be given estimating $S_{Z^t,Z^{t-4}}$ on the sample, by the formula:

$$\hat{S}_{Z^t,Z^{t-4}} = \frac{1}{n_c-1}\sum_{i\in s}(z_i^t g_i^t - \tilde{z}^t)\,(z_i^{t-4} g_i^{t-4} - \tilde{z}^{t-4}).$$

Knottnerus (2012) compares $Var(\hat{G}_{all})$ with $Var(\hat{G}_{olp})$. He finds the overlapping value (o) for which $Var(\hat{G}_{olp}) = Var(\hat{G}_{all})$. Above this value, the estimator $\hat{G}_{olp}$ performs better than the estimator $\hat{G}_{all}$. When we use calibration ($\hat{G}_{olp.call}$ and $\hat{G}_{all.cal}$ estimators), the procedure is the same used by Knottnerus, but the calculation must be made on the residuals of the generalized regression model. Sufficient condition for which $Var(\hat{G}_{olp.cal}) > Var(\hat{G}_{all.cal})$ is that $S_{Z^t,Z^{t-4}} < 0$ or $o < \frac{S_{Z^t-GZ^{t-4}}^2}{2GS_{Z^{t-4},Z^t}}$ provided that $S_{Z^{t-4},Z^t} > 0$.

4. Simulation study

A simulation study was conducted with the aim of analyzing the performance of these estimators. A population of N= 8360 units has been generated (without stratification) with turnover possessing a lognormal distribution with parameters (mean and variance) able to reproduce the population observed in the sector of Accommodation, in the size class between 2 and 5 employees. The population generated represents the universe at the occasion t-4. A calibration variable has been created according to the desired correlation with the interest variable Y^t. The created calibration variable has the same values for both occasions t and t-4. This make the simulation as similar as possible to the estimation process used for the estimation of the change in the service sector turnover in Istat. The sample size is calculated from the population at the occasion t-4, by means of the Bethel algorithms implemented in Mauss-R. The planned coefficient of variation for the estimation of the total turnover has been fixed at 3%. The result is a sample size of $n = 417$ units but a random non-response of 30% of the units in the sample has been applied in both occasions. This only serves to decrease the sample size and increase the variance of the growth rate estimation, to make it similar to what is likely to occur in the survey. The theoretical standard deviations of the turnover growth rate ($Se(\hat{g})$) were computed using each of the four estimators described in Section 2, together with: 1) Different correlation values (0.97, 0.92 and 0.86) between the study variable on the two survey occasions $Y^t$and Y^{t-4}. A higher/lower correlation is achieved by decreasing/increasing the variability of the data in Y^t. 2) Different values of the overlap (variable "o") between the units responding at the occasion t and the units responding at the occasion t-4. In particular, the results have been analyzed by considering overlapping of 5%, 10%, 15%, 20%, 25%, 30%, 50%, 70%, 99%. 3) Different values of the correlation between the variable of interest and the calibration variable. In particular, the results have been analyzed by considering correlation coefficient values *rho*=0, 0.5, 0.6, 0.7, 0.8, 0.9, 0.95, 1. For reasons of space, these values are reported only for correlation values on the study variable between t and t-4, equal to 0.97 and 0.86 (Table 1 and 2). Since $g = (G - 1) * 100$, we have that $Var(\hat{g}) = 100^2 Var(\hat{G})$ and then $Se(\hat{g}) = \sqrt{Var(\hat{g})}$. The tables also show the overlap thresholds (o) below which $Se(\hat{g}_{olp}) > Se(\hat{g}_{all})$ and $Se(\hat{g}_{olp.cal}) > Se(\hat{g}_{all.cal})$. For easier viewing in the tables, standard deviations below this threshold are colored in blue. As we can see from the results of the calculation of the standard deviations, when the overlap of the respondent units between the occasions increases, the standard deviation of all estimators decreases. This is in accordance with the sampling theory, because the variance of the change takes minimum value in the case of complete overlap (Kish, 1965, pp. 457-466). The standard deviations of the $\hat{g}_{all}$ and $\hat{g}_{olp}$ estimators are the same for each rho value because they do not need calibration. Using calibration we obtain the best results, therefore we have that $Se(\hat{g}_{all.cal}) \leq Se(\hat{g}_{all})$ and that $Se(\hat{g}_{olp.cal}) \leq Se(\hat{g}_{olp})$ for each rho ≠ 0 and for every overlap value. In particular, the greatest improvement is obtained when using the estimators based on all respondents ($\hat{g}_{all.cal}$ VS $\hat{g}_{all}$), while we observed only a limited improvement when using the estimators based on the overlap respondents ($\hat{g}_{olp.cal}$VS $\hat{g}_{olp}$).

Table 2 – *Standard deviation for the estimation of the growth rate g. Simulation 1:cor(x,y)=0.97.*

				rho=0				rho=0.7
overlap		calibration	no calibration			calibration	no calibration	
	Gall.cal	Golp.cal	Gall	Golp	Gall.cal	Golp.cal	Gall	Golp
0.05	6,6	5,1	6,6	5,2	4,7	5,0	6,6	5,2
0.10	6,4	3,7	6,4	3,6	4,6	3,6	6,4	3,6
0.15	6,3	3,0	6,3	3,0	4,6	2,9	6,3	3,0
0.25	5,9	2,3	5,9	2,3	4,3	2,3	5,9	2,3
0.30	5,7	2,1	5,7	2,1	4,1	2,1	5,7	2,1
0.50	4,9	1,6	4,9	1,6	3,5	1,6	4,9	1,6
0.70	3,8	1,4	3,9	1,4	2,8	1,3	3,9	1,4
0.99	1,3	1,1	1,3	1,1	1,2	1,1	1,3	1,1
O		0.03		0.03		0.06		0.03
				rho=0.9				rho=0.95
overlap		calibration	no calibration			calibration	no calibration	
	Gall.cal	Golp.cal	Gall	Golp	Gall.cal	Golp.cal	Gall	Golp
0.05	3,0	5,0	6,6	5,2	2,3	4,9	6,6	5,2
0.10	3,0	3,6	6,4	3,6	2,3	3,6	6,4	3,6
0.15	2,9	2,9	6,3	3,0	2,2	2,9	6,3	3,0
0.25	2,7	2,2	5,9	2,3	2,1	2,2	5,9	2,3
0.30	2,6	2,0	5,7	2,1	2,1	2,0	5,7	2,1
0.50	2,3	1,6	4,9	1,6	1,8	1,6	4,9	1,6
0.70	1,9	1,3	3,9	1,4	1,6	1,3	3,9	1,4
0.99	1,1	1,1	1,3	1,1	1,1	1,1	1,3	1,1
o		0.15		0.03		0.29		0.03

In this last case, the use of calibration leads to a smaller improvement because we have the same calibration variable (X) for both occasions (t and t-4) together with a low variability of X. In fact, in Table 2, where the variability of X is higher, the standard deviation values of $\hat{g}_{olp.cal}$ tend to be smaller than those of the $\hat{g}_{olp}$ estimator. When using calibration, to a higher rho value corresponds a higher overlap value over which $Se(\hat{g}_{olp.cal}) < Se(\hat{g}_{all.cal})$. This threshold also increases when the correlation between $Y^t$and Y^{t-4} decreases (with or without calibration). In fact, if we compare the tables, we can notice that the colored part becomes gradually larger in Table 2. The bias, the standard deviation and the mean squared error have been also analyzed through 1000 different samples extracted from the population. The absolute bias calculated from the 1000 estimates is very small. In fact, for most cases the bias is approximately equal to 0. For each estimate, a t-

Student distribution was used, and the corresponding 95% confidence intervals were calculated. The actual coverage probability of such confidence intervals is computed via simulation as the proportion of simulated confidence intervals that contain the true value of the growth rate g. As expected, the actual coverage probability is close to its nominal value, i.e. 95%. However, smaller values are obtained if the $\hat{g}_{olp}$ and the $\hat{g}_{olp.cal}$ estimators are used. In this case, especially for small overlap levels (5-10%), the coverage probability is approximately 90%. This is due to the fact that with low levels of overlap, the estimates were calculated on a small number of units (n_c). For example, with an overlap of 5%, only 15 units were used for the estimation. The simulation study was repeated in the case of stratified sampling design. Conditions very similar to those found in the ISTS were replicated: a level of overlap between the two occasions of about 70 percent, a high correlation between the variable of interest and the calibration variable (about 0.95) and a very high correlation between the observations on the two different occasions (about 0.98).

Table 3 – *Standard deviation for the estimation of the growth rate g. Simulation 3: cor(x,y)=0.86.*

				rho=0				rho=0.7
overlap		calibration	no calibration			calibration	no calibration	
	Gall.cal	Golp.cal	Gall	Golp	Gall.cal	Golp.cal	Gall	Golp
0,05	7.0	12.1	6.9	12.3	5.3	11.4	6.9	12.3
0,10	6.8	8.7	6.8	8.7	5.1	8.2	6.8	8.7
0,15	6.6	7.1	6.6	7.1	5.1	6.7	6.6	7.1
0,25	6.3	5.5	6.2	5.4	4.7	5.2	6.2	5.4
0,30	6.1	5.0	6.1	5.0	4.7	4.7	6.1	5.0
0,50	5.4	3.9	5.3	3.8	4.2	3.6	5.3	3.8
0,70	4.5	3.3	4.4	3.2	3.7	3.1	4.4	3.2
0.99	2.8	2.7	2.8	2.7	2.6	2.6	2.8	2.7
o		0.17		0.17		0.30		0.17

				rho=0.9				rho=0.95
overlap		calibration	no calibration			calibration	no calibration	
	Gall.cal	Golp.cal	Gall	Golp	Gall.cal	Golp.cal	Gall	Golp
0.05	3.7	11	6.9	12.3	3.1	10.8	6.9	12.3
0.10	3.7	7.8	6.8	8.7	3.1	7.8	6.8	8.7
0.15	3.6	6.4	6.6	7.1	3.1	6.3	6.6	7.1
0.25	3.5	4.9	6.2	5.4	3.0	4.9	6.2	5.4
0.30	3.4	4.5	6.1	5.0	2.9	4.4	6.1	5.0
0.50	3.2	3.5	5.3	3.8	2.8	3.4	5.3	3.8
0.70	2.9	3.0	4.4	3.2	2.7	2.9	4.4	3.2
0.99	2.5	2.4	2.8	2.7	2.4	2.4	2.8	2.7
o		0.74		0.17		1.0		0.17

Table 3 contains the summary statistics about the generated population for the occasion t and t-4. The coefficient of variation needed to calculate the sample size is set at 3 percent for the total estimation domain (not within each stratum). The bias and the standard deviation have been also analyzed through 300 different samples extracted from the population. Compared with the previous simulation fewer replications were made since the population is larger (N=19,889). As we can see from Tables 4 and 5, the estimators have a strong bias and standard deviation within the strata. Stratum 4 is an exception, because it is a census stratum. Instead, within the estimation domain the bias is nearly 0 for all the estimators except for the estimator G_{all} (1.1 p.p.). Standard deviations within the estimation domain are smaller than the ones within the strata. The best estimators are $\hat{G}_{olp.cal}$ and $\hat{G}_{olp}$. For these estimators, the mean squared error within the estimation domain is the same. This is probably due to the low variability of the calibration variable within the strata, which makes the calibrated weights very similar to the design weights.

Table 4 – *Summary statistics of the simulation in case of stratification of the population*

Strata	N	n	Samplinng fract.%	nr	o	nc	Cor(Yt,Yt-4)	Growth rate g%
1	8,413	30	0.4	21	1	14	0.98	-10.1
2	9,885	140	1.4	98	1	69	0.97	-9.8
3	1,456	83	5.7	58	1	41	0.98	-9.6
4	135	135	100	95	1	66	0.95	-9.2
Total	19,889	388	2	272	1	190	0.98	-9.7

Table 5 – *Bias (p.p) and SD calculated on 300 sample estimates for the growth rate g.*

Stratum/Domain	calibration		no calibration	
Bias	Gall.cal	Golp.cal	Gall	Golp
		Bias		
Stratum1	1.5	-0.4	4.7	-0.5
Stratum2	0.1	-0.2	0.7	-0.2
Stratum3	0.4	0.1	1.6	0.2
Stratum4	0	0	0	0
Domain	0.2	-0.1	1.1	-0.2
		SD		
Stratum 1	14.7	5.7	26.2	5.9
Stratum 2	4.2	2.7	8.7	2.8
Stratum 3	5.4	3.1	11.2	3.2
Stratum 4	2.2	1.8	5.2	1.8
Domain	2.8	1.5	5.4	1.5

5. An application to the service turnover survey data

The application was performed on 2 different domains corresponding to two different economic activities. The first domain (D1) consists of four different estimation domains (G1, G2, G3, G4). The second domain (D2) consists of two different estimation domains (G5 and G6). Each domain estimation (G1, G2, G3, G4, G5 and G6) is divided into four independent strata according to the class of employees, with the exception of one estimation domain (G1), which is instead divided into three independent strata. The stratum 4 (with more than 100 employees) within each estimation domain is the self-representative stratum. The application has been conducted on a given estimation quarter (which is not specified here). The estimators used for the growth rate estimation are those described in the previous chapter ($\hat{g}_{d,olp}$, $\hat{g}_{d,all.cal}$, $\hat{g}_{d,olp.cal}$). Since, as seen from the simulation study, the estimator $\hat{g}_{d,all}$ gives the worst results in terms of standard error of the growth rate estimation, it has not been used in the present application. The sample correlation between the variable of interest and the calibration variable (rho) is very high (0.99 for the domain D1 and 0.96 for the domain D2). Standard errors have been calculated using the Taylor series approximation. These values were compared with those obtained using the bootstrap method. Using the method proposed by Holmberg (1998), three artificial stratified populations (U_t^*, $U_{t,t-4}^*$ and U_{t-4}^*) were created and 300 bootstrap samples were generated from the artificial resampling populations in such a way that the overlapping of the units between the two quarters is the same as the parent sample, within each stratum.

Table 6 – *Standard error of the growth rate estimation for some estimation domains.*

Domain/Group	Overlap	Taylor series Approximation			Bootstrap method		
		$\hat{S}e$ $(\hat{g}_{olp})$	$\hat{S}e$ $(\hat{g}_{all.cal})$	$\hat{S}e$ $(\hat{g}_{olp.cal})$	$\hat{S}e$ $(\hat{g}_{olp})$	$\hat{S}e$ $(\hat{g}_{all.cal})$	$\hat{S}e$ $(\hat{g}_{olp.cal})$
G1	0.84	1.4	1.3	1.2	1.1	1.1	1.0
G2	0.78	1.4	1.3	1.3	1.3	1.2	1.3
G3	0.82	1.1	1.0	0.7	0.8	0.7	0.7
G4	0.74	1.2	1.3	1.0	1.1	1.2	1.1
D1	**0.79**	**1.0**	**0.9**	**0.8**	**0.8**	**0.8**	**0.7**
G5	0.72	0.9	1.9	0.9	0.8	1.7	0.9
G6	0.70	0.7	1.7	0.7	0.6	1.4	0.7
D2	**0.71**	**0.8**	**1.5**	**0.7**	**0.7**	**1.4**	**0.7**

The results for the standard error are shown in Table 6. The results obtained with the bootstrap method in terms of standard errors are quite close to those obtained with the Taylor series approximation. Observing the results obtained through the Taylor series approximation, the best results are obtained with the use of the estimator $\hat{g}_{olp.cal}$.

6. Conclusion

The simulation study and the application show that given the characteristics of the ISTS, the estimator with the smallest standard errors is the calibration estimator calculated on the overlapping sample units in both quarters. The mentioned characteristics are: a high overlapping level (above 70%), a high correlation between the variable of interest and the calibration variable (greater than 0.95) and a very high correlation between the observations in the two occasions. In addition, using the computed standard errors, it was possible to calculate a confidence interval associated with the change in turnover in some of the estimation domains for the ISTS, allowing the accuracy of the estimate produced to be measured.

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SUMMARY

The aim was to compute the variance of the estimators currently used in the service turnover survey for the quarterly turnover growth rate estimation and identify the best estimator. The survey uses two indicators for the estimation of the growth rate. The first one is a ratio between two mean estimators and is calculated on the set of respondents common to both quarters ($\hat{G}_{olp}$). The second estimator is instead the ratio between two totals in two different occasions, calculated using the calibration estimator. This second estimator is applied to the whole set of respondents in both periods ($\hat{G}_{all.cal}$). Since both estimators are non-linear function of linear estimators, the first-order Taylor approximation was used to compute the variance. To identify the best estimator, a simulation study has been conducted: two populations referred to two different occasions were generated and 1,000 samples were extracted. Therefore, it was possible to compute the bias, the standard deviation and the mean squared error for the estimation of the turnover growth rate. The analysis was performed for different sample overlapping values between the two reference quarters and different correlation values between the variable of interest and the calibration variable, together with different correlations of the variable of interest between the two occasions. An application performed on real data was also conducted, using information from the quarterly service turnover survey. The confidence intervals associated with the year-over-year variation of the quarterly service turnover were calculated for some estimation domains. The standard errors obtained by using Taylor first-order series approximation were compared with the ones obtained with the bootstrap method. The comparison shows similar results.

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